Trabalho de um campo vectorial

(Integral de linha de um campo vectorial).

Campos fechados:

$F: \mathbb{R}^n \to \mathbb{R}^n$, C^1

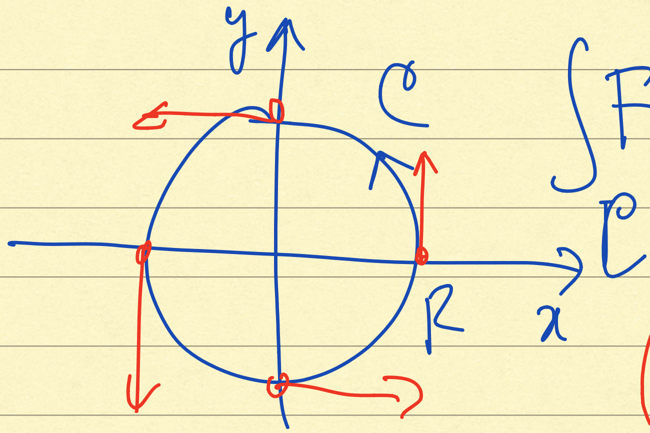
$$\frac{\partial F_j}{\partial x_k} = \frac{\partial F_k}{\partial x_j} \qquad j \neq k$$

$$\boxed{\text{Se } F = \nabla \varphi \text{ então } (\Longrightarrow) \ F \text{ é fechado}}$$

$\varphi: \mathbb{R}^n \to \mathbb{R}$, C^1

Exemplo: 1) $F(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$

$(x,y) \neq (0,0)$



$$\int_C F\cdot d\gamma = 2\pi \quad \forall R>0$$

(1 volta)

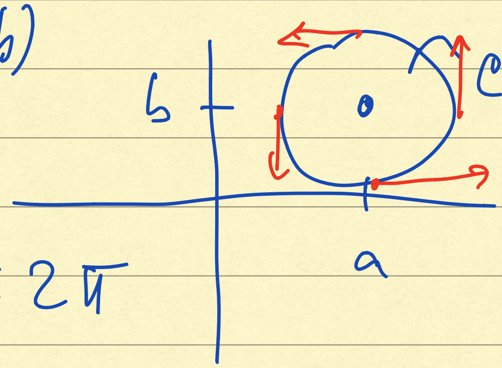
Exercício: N voltas $\int_C F\cdot d\gamma = 2\pi N$.

$$\boxed{\frac{1}{2\pi}\int_C F\cdot d\gamma = N}$$

2) Sinal !!!

$$F(x,y) = \left(-\frac{y-b}{(x-a)^2+(y-b)^2},\ \frac{x-a}{(x-a)^2+(y-b)^2} \right)$$

$(x,y) \neq (a,b)$

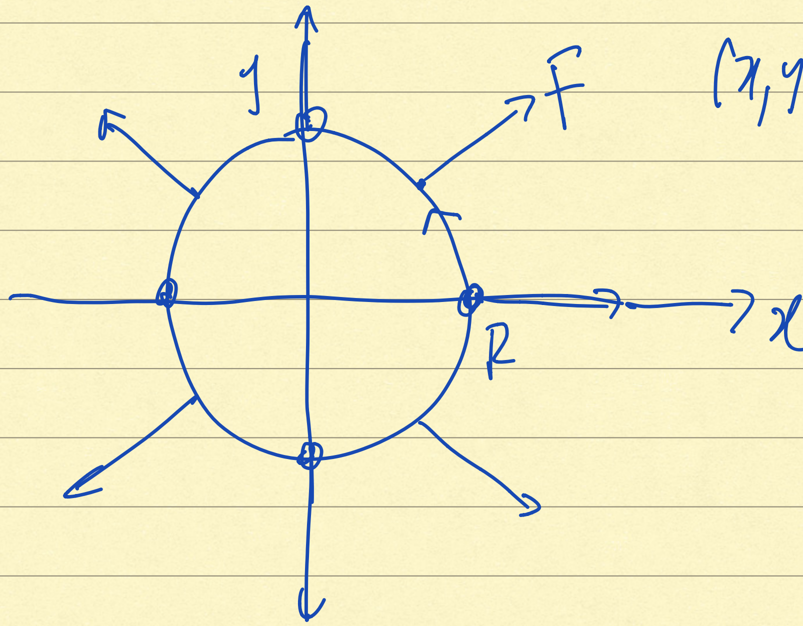


1 volta

$$\int_C F\cdot d\gamma = 2\pi$$

$$(x-a)^2+(y-b)^2 = R^2$$

3) $F(x,y) = \left(\dfrac{x}{x^2+y^2} , \dfrac{y}{x^2+y^2} \right)$



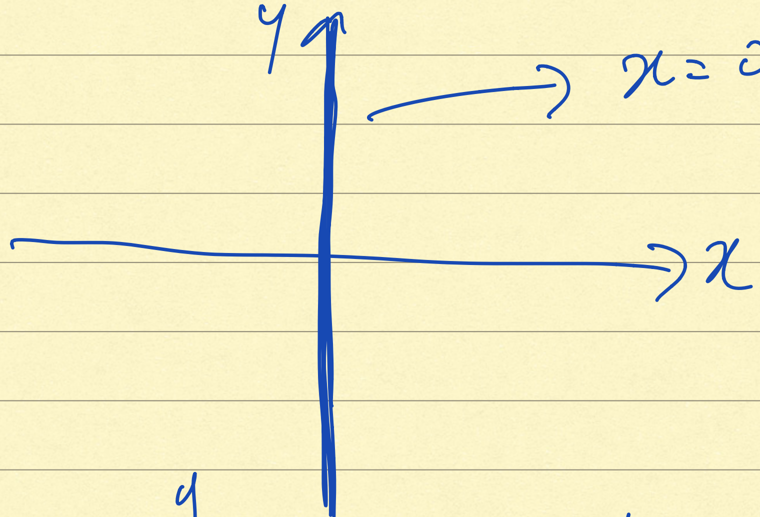
$(x,y) \neq (0,0)$

$F = \nabla \varphi \quad ; \quad \varphi(x,y) = \frac{1}{2} \log(x^2+y^2) + C$

— || —

4) $\varphi(x,y) = \arctan\left(\dfrac{y}{x}\right)$

$x \neq 0$ (eixo Oy)



$$\frac{\partial \varphi}{\partial x} = \frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2} = -\frac{y}{x^2+y^2}$$

$$\frac{\partial \varphi}{\partial y} = \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2} = \frac{x}{x^2+y^2}$$

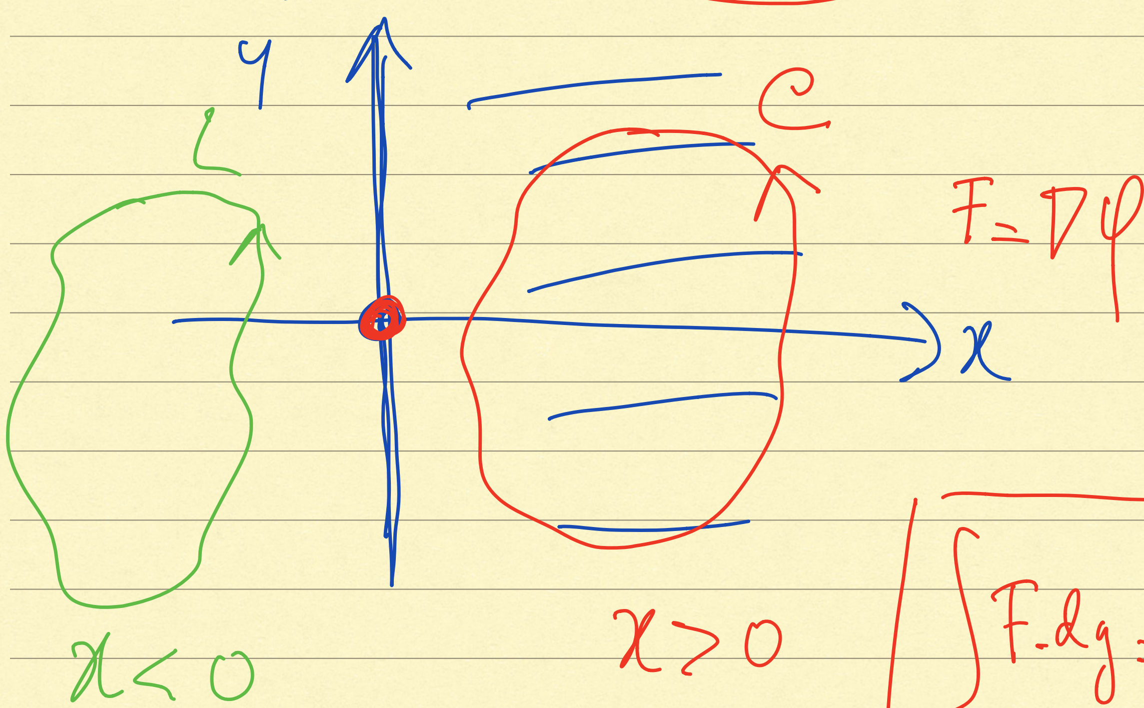
$$\nabla \varphi(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right) =: F(x,y)$$

$$\varphi : \mathbb{R}^2 \setminus \{(0,y)\} \longrightarrow \mathbb{R}$$

$$F : \mathbb{R}^2 \setminus \{(0,0)\} \longrightarrow \mathbb{R}^2$$

Domínio de $\varphi \neq$ domínio de F!

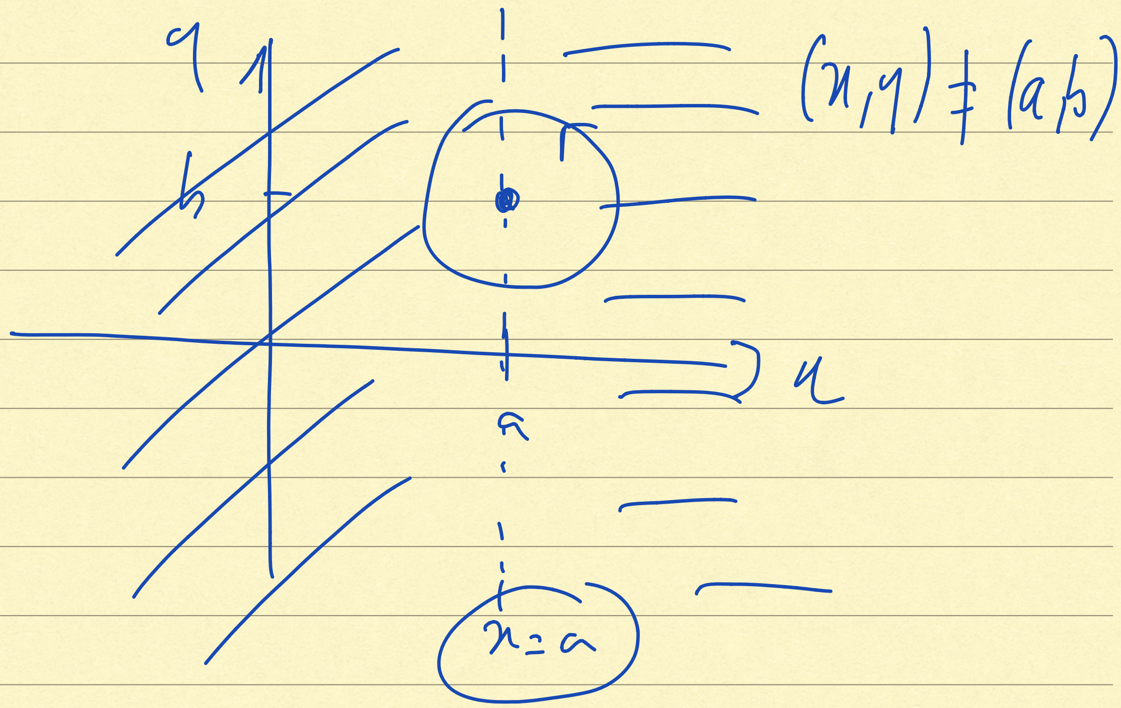
Apesar de F não ser gradiente em $\mathbb{R}^2 \setminus \{(0,0)\}$, é gradiente em $\{(x,y) : x > 0\}$.



$$\boxed{\int_{C} F \cdot d\gamma = 0}$$

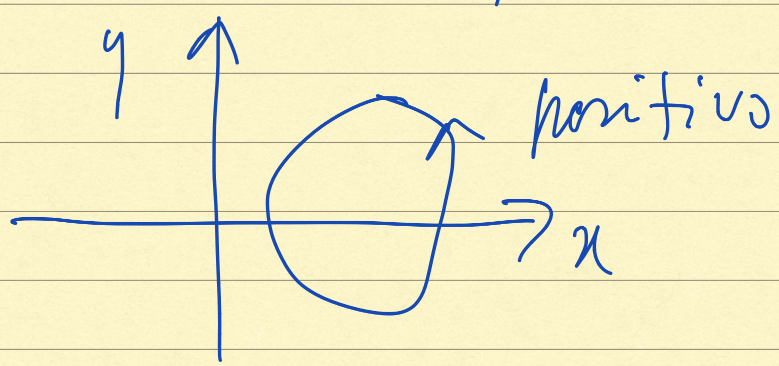
L

$$F(x,y) = \left(-\frac{y-b}{(x-a)^2+(y-b)^2},\ \frac{x-a}{(x-a)^2+(y-b)^2} \right)$$

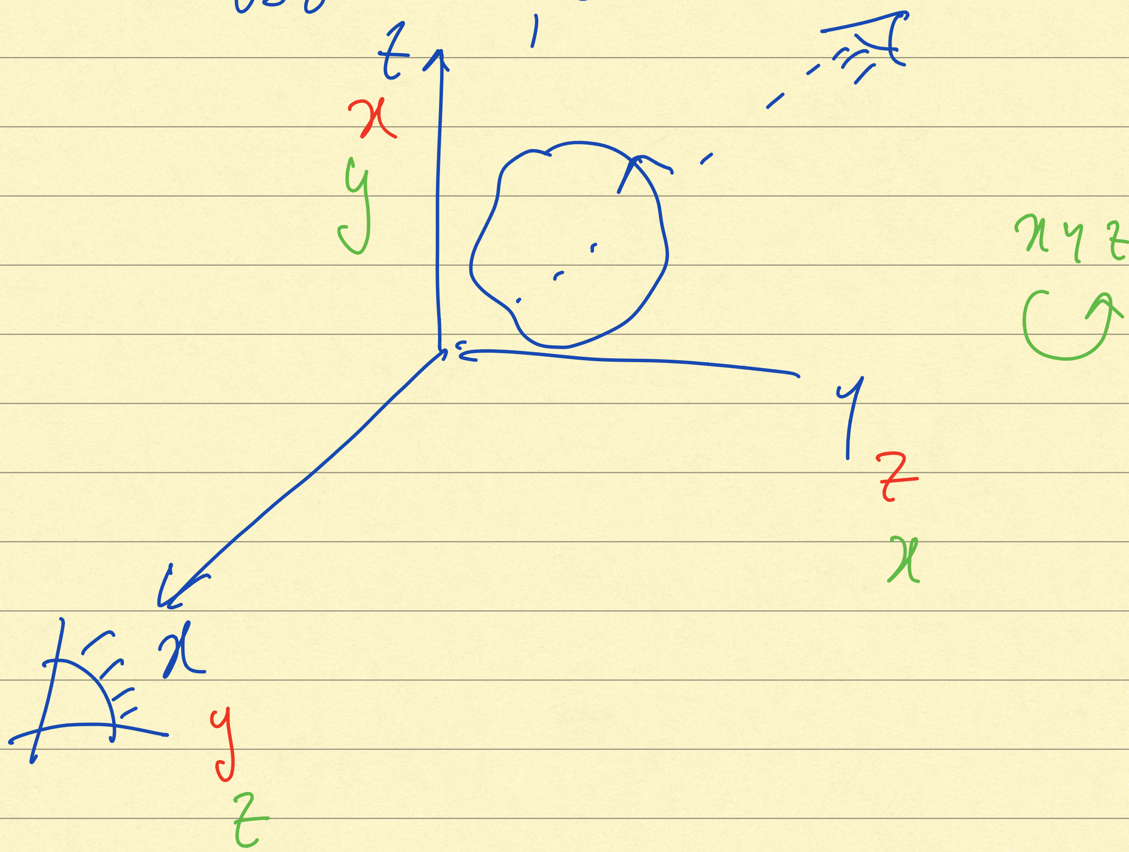


Dúvida: orientação da linha

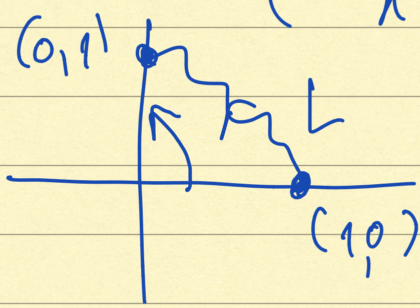
$\mathbb{R}^2$:



$\mathbb{R}^3$: Depende do ponto de observação da linha.



$$F(x,y) = \left(-\frac{y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$$



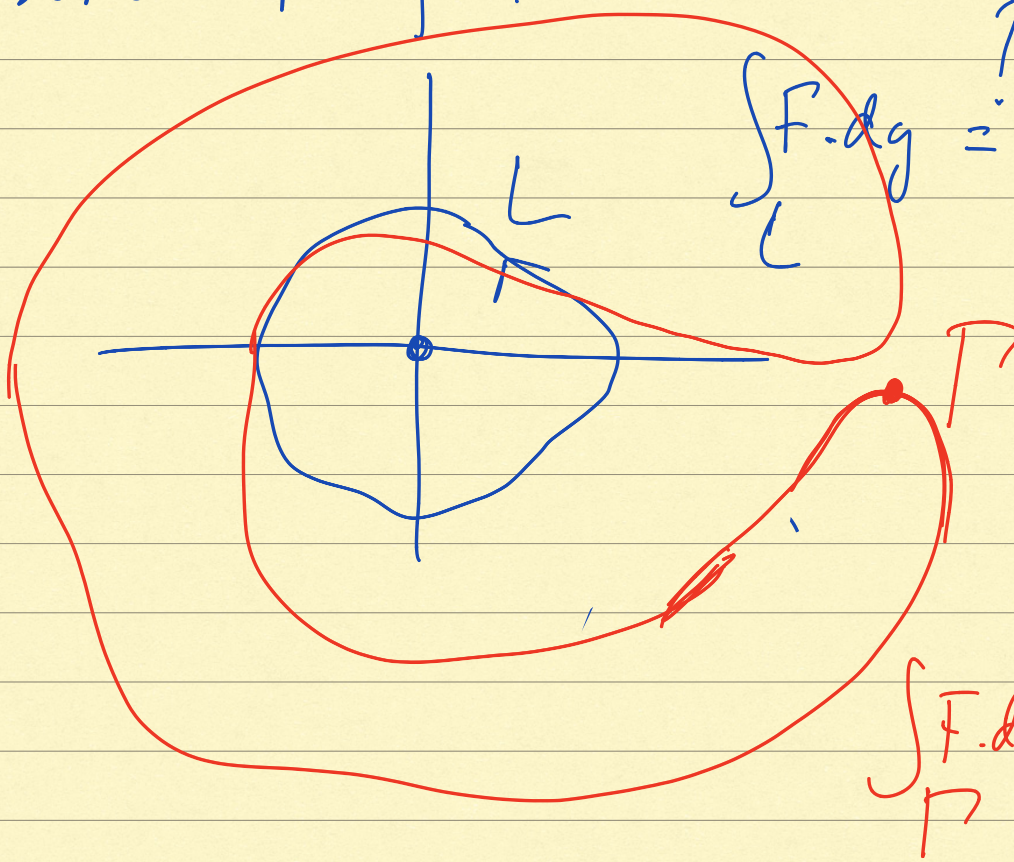
$$\int_L F \cdot d\gamma = ?$$

$$\int_L F = \varphi(0,1) - \varphi(1,0)$$

$$= \arctan(\infty) - \arctan(0)$$

$$= \frac{\pi}{2}.$$

Some F fate zahen :



$$\int_L F \cdot d\gamma \overset{?}{=} 2\pi$$

$$\int_\Gamma F \cdot d\gamma \overset{?}{=} 0$$